CALCULATION OF THE THERMAL INTERACTION BETWEEN AN ARC COLUMN AND THE ELECTRODES

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The temperature distribution in an arc column of finite length is treated as a two-dimensional nonlinear boundary-value problem with the cooling effect of the electrodes taken into account. The sharp temperature drop near the electrodes is represented by a thermal boundary layer of the arc, without explicitly considering the structure of the zones adjacent to the electrodes. The temperature, the thermal fluxes into the electrodes, and the field intensity as well as the voltage in the arc are calculated.

In the theory of an arc-discharge column stabilized by walls no consideration is usually given to its thermal interaction with the electrodes. Only energy losses in the radial direction are accounted for, the column is considered sufficiently long and lengthwise homogeneous [1]. One further assumes that the near-electrode regions, namely the cathode and the anode fall, separate the column from the electrodes and that the latter interact with those narrow potential fall zones only. From the point of view of interaction kinetics and plasma–solid transition, this strict but predominant interaction between the electrodes and the zones adjoining them remains unquestionable. However, the temperature drops between the column and the electrodes reach several thousand degrees. At the electrode segments the temperature is maintained near the melting point somewhere within 2000-4000°K while the temperature in the column reaches 5000-15,000°K or higher, depending on the current, on the medium, and on other conditions under which the arc is sustained. The existence of such a large temperature drop must result in an intensive heat flow to the electrodes and, consequently, the cylindrical symmetry in the regions adjoining the electrodes must become considerably distorted. Large thermal fluxes entering the electrodes have, indeed, been observed experimentally and have been estimated by the rough formulas $Q_T = i(\Delta V_\nu \mp \varphi)$, which do not indicate, however, how these thermal fluxes depend on the current, on the gas and the gas pressure, or on the geometry of the discharge gap [2], because the magnitudes of the voltage drops ΔU_ν in the near-electrode regions are determined by test. The sharp distinction between the near-electrode zones and the arc column is usually emphasized. Under prevailing large temperature drops between the column and the electrodes, however, the narrowness of the near-electrode regions (it can be measured in terms of the free electron-flight path) should cause the cooling effect of the electrodes on the column to spread over wider distances and the thermal interaction between the column and the electrodes to be more precise than had been considered before. From the point of view of temperature distribution along the discharge axis, the near-electrode zones may be treated as a thermal boundary layer. The thermal interaction between the column and the electrodes can then be studied, to a first approximation, without accounting for process kinetics in the near-electrode zones. The sharp decrease in temperature and in conductivity near the electrodes must lead to a sharp increase in field intensity, to ensure that the total discharge current be maintained constant in the near-electrode zones where the voltage drops are correspondingly large. Actual arc, especially high-power arcs, are often not more than a few centimeters long and the effect of the electrodes may in this case be particilarly significant. Consideration of the cooling effect of the electrodes on the energy balance in the column may be expected to yield a better agreement between calculated and observed column temperatures, especially at medium and heavy currents, to make it possible to calculate the thermal fluxes entering the electrodes and to calculate the electrodes wear, also to yield a more accurate calculation of the total arc voltage for arcs of finite length.

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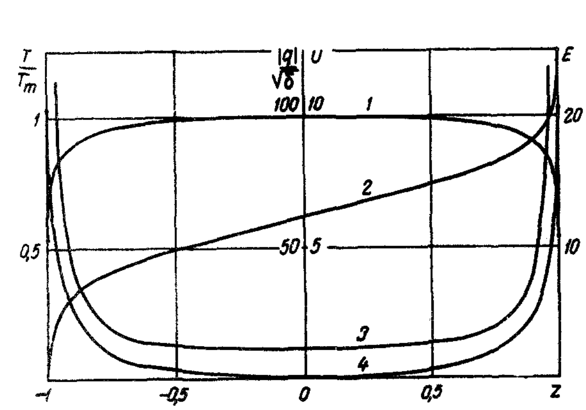

Fig. 1

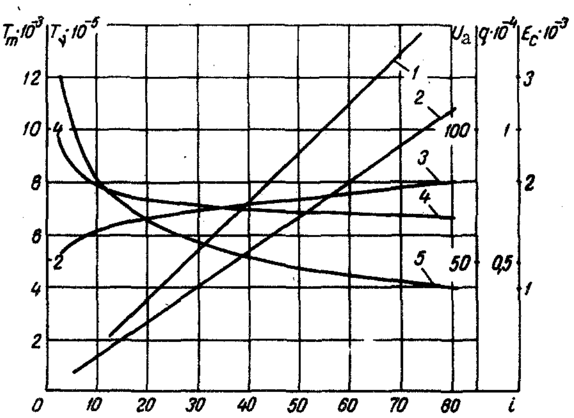

Fig. 2

Fig. 1. Distributions along the discharge axis with $i/r_w = 3.1 \cdot 10^3$ A/m and $\delta_1 = 0.18$: 1) temperature T/T_m; 2) potential U; 3) field intensity E, and 4) thermal flux q/δ_1.

Fig. 2. Discharge parameters as function of the current at $\delta_1 = 0.18$ and $r_w = 10^{-2}$ m: 1) thermal flux into an electrode at the axis, q_ν (kW/m^2); 2) field intensity at the electrode $E_{c\nu}$ (V/m); 3) maximum temperature, T_m (°K); 4) voltage in the arc, U_d (V); 5) field intensity at the center of column E_{cc} (V/m).

We will analyze the column of an arc discharge under constant pressure along the z'-axis of a tube with the radius r_w and the distance l_d between the electrodes. Taking into consideration the radial as well as the axial heat flow (disregarding gravity and, therefore, convection), we will write the equation of heat balance with respect to the thermal potential $S = \int_0^T \lambda dT$ and Ohm's law:

$$\frac{1}{r'} \cdot \frac{\partial}{\partial r'}\left(r' \frac{\partial S}{\partial r'}\right) + \frac{\partial^2 S}{\partial z'^2} + \sigma_c E_c^2 - Q_i = 0, \tag{1}$$

$$i = \int_0^{r_w} E_c \sigma_c 2\pi r' dr'. \tag{2}$$

The radiation losses per unit volume $Q_i(S)$ do not include here reabsorption, and the field intensity in the column $E_c(z')$ is assumed constant over its cross section. The temperature at the wall of the discharge tube will assumed known and this, together with the finiteness of temperature at the axis, yields the following conditions:

$$S(r'_w, z') = S_w; \qquad \frac{\partial S(0, z')}{\partial r'} = 0. \tag{3}$$

An essential difficulty here is the formulation of the boundary conditions at the electrodes. The actual boundary conditions for the temperature at the end sections of the electrodes must include several factors: different process parameters at the cathode and at the anode, incomplete coverage of the electrode tips with basal spots, and dependence of the area covered on the current, on the temperature distribution across a tip surface as well as across the region between electrode and wall, etc. A precise accounting of all these effects is difficult, and approximately defined boundary conditions may be adequate for calculation purposes. In order to formulate them, we will assume that the temperature at the centers of the discharge spots on an electrode – on the cathode as well as on the anode – is equal to the melting point of the electrode material. For the tip sections of the electrodes one may approximate function S as follows:

$$S(r', z'_\nu) = S_w + (S_\nu - S_w)\psi_\nu\left(\frac{r'}{r_w}\right), \tag{4}$$

where S_ν are the values of S at the centers of electrode spots, ψ_ν is the value of a function which decreases monotonically from the axis to the wall along a profile determined by the process parameters of the electrode: $\psi_\nu(0) = 1$, $\psi_\nu(1) = \psi'_\nu(0) = 0$. Such a statement of the boundary conditions at the electrode tip surface is admissible for electrodes without forced cooling. When the electrodes are cooled intensively (e.g., with water), the temperature of the electrode spots is unknown and it becomes necessary to establish a condition of the third kind. Various kinds of boundary conditions may be given at the anode and at the cathode. In the configuration considered here we do not account for the evaporation of electrode material and for changes in the plasma composition due to an injection of electrode material, also not for radiation from the electrode surface, etc. Energy for the evaporating of the electrode material also comes from the discharge column. Since it is not included in the boundary condition (4), this should make the thermal fluxes from the arc column to an electrode tip surface appear smaller with apparently little effect, however, on the thermal fluxes entering the bulk of the electrodes. A more precise statement pertaining to the interface of the arc column with an electrode would require that the boundary condition be expressed in a nonlinear form [3]:

$$-\lambda\frac{\partial T(r',\ z'_\nu)}{\partial z'}=\lambda_\tau\ \mathrm{grad}\ T+\lambda_{ev}G(T)+\varkappa T^4. \tag{4'}$$

This report will be limited to an approximate solution of the problem as stated by Eqs. (1)-(4), which will yield significant results even though condition (4) is only approximate.

The fundamental equations and the boundary conditions will be converted into a dimensionless form, with the origin of coordinates at the center of the column between the electrodes:

$$\delta\frac{\partial^2 y}{\partial z^2}+\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial y}{\partial r}\right)+\sigma(y)E^2(z)-\beta Q(y)=0, \tag{5}$$

$$1=E(z)\int_0^1\sigma r dr, \tag{6}$$

$$y(1,\ z)=\frac{\partial y(0,\ z)}{\partial r}=0, \tag{7}$$

$$y(r,\ \pm 1)=\alpha_\nu\psi_\nu(r), \tag{8}$$

where $\delta = 4r_w^2/l_d^2$, $y = (S-S_w)/S_*$, $r = r'/r_w$, $z = 2z'/l_d$, $\alpha_\nu = (S_\nu - S_w)/S_*$, $\sigma = \sigma_c/\sigma_*$, $E = E_c/E_*$, $\beta = Q_* r_w^2/S_*$, and $Q = Q_i/Q_*$. The quantities S_* and E_* are determined from the ratios

$$\frac{\sigma_* E_*^2 r_w^2}{S_*}=1;\quad \frac{i}{E_*\sigma_* 2\pi r_w^2}=1,$$

while σ_* and Q_* are determined after the introduction of specific functions $\sigma_c(S)$ and $Q_i(S)$.

The presence of the small diameter δ with the second derivative in (5) confirms the existence of boundary layers near the electrodes [4]. In addition to the nonlinearity of Eqs. (5) and (6), this creates great difficulties in solving our two-dimensional boundary-value problem. Considering that of most interest in this report is the temperature distribution along the discharge axis, we will find an approximate solution to the problem (5)-(8) by the direct method or reducing it to ordinary equations and stopping at the first approximation, as was done in [5]. We will thus seek the approximate solution in the form $y = a(z)\cdot\psi(r)$, where $\psi(r)$ is the given function which satisfies conditions (7) and $a(z)$ is a new unknown function. In order to determine the latter, we eliminate E from (5) by means of (6) and then average (5) as well as (8) over the cross section of the discharge tube. We obtain an ordinary differential equation and the boundary conditions for $a(z)$:

$$\delta_1\frac{d^2a}{dz^2}-a+\frac{1}{\bar{\sigma}(a)\,|\psi'(1)|}-\beta_1\bar{Q}(a)=0, \tag{9}$$

$$a(\pm 1)=\alpha_{\nu 1}, \tag{10}$$

where $\delta_1=\dfrac{4r_w^2 m_1}{l_d^2|\psi'(1)|}$; $\bar{\sigma}(a)=\int_0^1\sigma r dr$; $\bar{Q}(a)=\int_0^1 Q r dr$; $\beta_1=\dfrac{Q_* r_w^2}{S_*|\psi'(1)|}$; $\alpha_{\nu 1}=\dfrac{\alpha_\nu m_{\nu 1}}{m_1}$; $m_{\nu n}=\int_0^1\psi_\nu^n r dr$; $m_n=\int_0^1\psi^n r dr$.

The solution to the boundary-value problem (9)-(10) is found in quadratures.

The resolution of $a(z)$ is determined from the equation:

$$\frac{z-z_m}{\sqrt{\delta_1}} = \pm \int\limits_{a_m}^{a} \frac{da}{\sqrt{F(a)-F(a_m)}}, \tag{11}$$

where $F'(a) = 2f(a) = a - 1/(\bar{\sigma}(a)|\psi'(1)|) + \beta_1\bar{Q}(a)$, and $a_m(z_m)$ is the maximum value of $a(z)$ on the interval $(-1, 1)$ determined together with z_m from the system of equations:

$$\frac{z_m+1}{\sqrt{\delta_1}} = \int\limits_{\alpha_{a1}}^{a_m} \frac{da}{\sqrt{F(a)-F(a_m)}}; \quad \frac{z_m-1}{\sqrt{\delta_1}} = -\int\limits_{\alpha_{k1}}^{a_m} \frac{da}{\sqrt{F(a)-F(a_m)}}. \tag{12}$$

The thermal flux along the column axis is determined from the equation

$$q_c(z) = \pm q_* \sqrt{F(a)-F(a_m)}, \tag{13}$$

where

$$q_* = \frac{S_*}{r_w}\sqrt{\frac{|\psi'(1)|}{m_1}}.$$

When $a = \alpha_{\nu 1}$, Eq. (13) yields the thermal fluxes going into the electrodes. They increase slightly as the arc becomes longer, till $q_\nu \to q_{max}$ for $l_d \to \infty$; that $\delta_1 \to 0$, is explained by the higher temperature gradients at higher temperatures T_m, resulting from a reduced cooling effect of an electrode. Also $a_m \to a_0$, where a_0 is the solution to (9) at $\delta_1 = 0$. For all practical purposes, the thermal fluxes into the electrodes do not significantly depend on a_m, and for not very short arcs $(2r_w/l_d \le 1)$ one may replace q_ν by q_{max} in the calculations. Thus,

$$q_{max} = q_* \sqrt{F(\alpha_{\nu 1}) - F(a_0)}. \tag{14}$$

The distributions of field intensity and potential along the discharge axis are defined by the expressions

$$E(z) = \frac{1}{\bar{\sigma}(a)}; \quad U(z) = \int\limits_{-1}^{z} E(z)\,dz,$$

with $U_* = E_* l_d$.

In order to reveal the effect of accounting for the electrodes, we will show the results of numerical calculations performed for the specific case where the electrical conductivity is approximated by a power-law relation: $\sigma_c = \sigma_n (S - S_w)^n$ with radiation disregarded with symmetrical boundary conditions assumed at the electrodes. Such boundary conditions occur approximately in arcs without a cathode spot, when one may assume that $\psi_a \equiv \psi_c \equiv \psi = J_0(\mu_1 r)$ with $\mu_1 = 2.405$ and $S_c = S_a$. (For arcs with a cathode spot one may assume, for example,

$$\psi_c = \frac{\exp(-\gamma r^2) - \exp(-\gamma)}{1 - \exp(-\gamma)},$$

where the parameter γ is determined by the current density at the electrode spots.) For nitrogen at atmospheric pressure and graphite electrodes one may assume $n = 2$; $\sigma_2 = 10^{-5}$ m/Ω·W, $S_* = 2.93 \cdot 10^2 (i/r_w)^{2/3}$ W/m; $S_\nu = 4.68 \cdot 10^2$ W/m, and $\alpha_\nu = 0.883$; $S_w \ll S_\nu$.

The axial distribution of T, E, q, and U are shown in Fig. 1. The boundary layer at each electrode is clearly in evidence here, in terms of temperature as well as in terms of field intensity and voltage. The dependence of the basic quantities on the current is shown in Fig. 2. The presence of a boundary layer results in a modification of the field–current characteristic along the discharge axis. While the field vs current curve is falling in arc regions far from an electrode, it becomes rising near the electrodes. This is explained by the temperature and the conductivity at the electrode surfaces, both determined by the boundary conditions, being independent of the current. The existence of column sections with a rising field–current characteristic corresponds to the appearance of a rising branch in the volt–ampere curve, which does not appear when $\delta_1 = 0$ with a power-law approximation of $\sigma_c(S)$. The location of the minimum on the volt–ampere curve is approximated by the equation: $U_d = U_* [U_0 + U_1 \delta_1^m (i/r_w)^n]$ and depends on δ_1. For the values listed earlier, $U_0 = 2.26$; $U_1 = 2.56$; $m = 0.583$, and $n = 0.417$. The voltage across the arc and the amplitudes of the near-electrode voltage drops, which are obtained from the potential curve by extending the linear portion of the curve all the way to the electrodes, agree sufficiently well with the test values in [1].

The solution corresponding to the original system (1)-(4) is asymptotic with respect to the arc current, since the boundary conditions at the electrodes become invalid at $i \to 0$. At the same time, the temperature distribution in the arc discharge with even a small arc current is very different from that distribution with no arc current and is adequately well described by this system of equations. This comment applies also to the thermal fluxes.

With the specific values given earlier, q_{max} increases almost linearly with the arc current and realistic values are obtained for the thermal fluxes beginning at 1-2 A or heavier currents.

It was not possible to compare the calculated thermal fluxes into the electrodes with experimental data directly, because the thermal flux measurements were not available to the author. The measurements for vacuum arc are given in [6] and the character of the relation (linear) as well as the magnitudes of the thermal fluxes agree closely with calculations according to formula (14). The comparison with the data in [6] must be not considered entirely valid, however, since the calculations were made for nitrogen at atmospheric pressure. It would be interesting to measure the thermal fluxes into the electrodes according to the procedure in [6] for arcs in air.

The obtained results show that the approach proposed here allows one to calculate several basic properties of an arc discharge whose length is finite, as observed in practice, without going into detail as to the kinetic pattern near the electrodes.

NOTATION

r', z' cylindrical coordinates;
r_w tube radius;
l_d arc length;
i current;
E_c field intensity in the column;
T temperature;
λ thermal conductivity of plasma;
σ_c electrical conductivity;
S thermal potential;
Q_i energy loss due to radiation, per unit volume;
Q_T total thermal flux;
q specific thermal flux;
ΔU near-electrode potential drop;
φ work function;
λ_T thermal conductivity of electrode material;
λ_{ev} evaporation heat of electrode material;
a dimensionless thermal potential at the tube axis;
$\psi(r)$ S-distribution function over the cross section.

Subscripts

w denotes condition at the wall;
$\nu = c, a$ conditions at the cathode and the anode respectively;
* indicates the fundamental magnitude;
c conditions in the column;
m point of maximum a;
0 conditions in a discharge without electrodes.

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